The whitening of ambient seismic noise

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***Abstract*—Whitening of recordings of ambient seismic noise prior to crosscorrelation is widely employed. We evaluate how the real part of the cross-spectrum behaves as a function of distance for whitened recordings. We show that it is proportional to a Bessel function for distances larger than approximately one fourth of a wavelength.**

I. Introduction

Over the last decade the field exploiting ambient seismic noise has rapidly developed. Much attention has been paid to the preprocessing of data, which includes spectral whitening. Whitening of the spectra prior to cross-correlation turns out to be very effective (Seats et al., 2012). Researchers estimating attenuation based on seismic interferometric (SI) measurements of surface waves also employ spectral whitening (Lawrence and Prieto, 2011; Weemstra et al., 2012).

The methodology of these attenuation studies is based on the derivation of the normalized spatial autocorrelation (SPAC) by Aki (1957). He shows that, given a stationary wavefield over a laterally homogenous medium, the azimuthally averaged cross-spectrum coincides with a Bessel function. Investigators estimating subsurface attenuation generally fit a damped Bessel function to the real part of the normalized azimuthally averaged coherency (Lawrence and Prieto, 2011; Weemstra et al., 2012).

An apparent inconsistency between their results and the theoretical framework associated with the SPAC-method appears however: they require the real part of the normalized azimuthally averaged coherency to be fit by a downscaled version of the damped Bessel function instead of a damped Bessel function (Lawrence, 2012; Weemstra et al., 2012). We show that this discrepancy is due to the difference between the normalization employed by the SPAC-method and the implicit normalization associated with spectral whitening.

II. Theory

We define the frequency domain crosscorrelation for recordings $u(\boldsymbol{x})$ and $u(\boldsymbol{y})$, captured at surface locations $\boldsymbol{x}$ and $\boldsymbol{y}$, as $\hat{C}_{xy}(\omega)$, where ω is the angular frequency. The expression used for calculating the SPAC (e.g. Aki, 1957; Okada, 2003) is dubbed the "averaged complex coherency" and is obtained by azimuthal averaging of complex coherencies associated with individual station pairs. For an individual station pair the complex coherency, ρ, is defined,

$$\rho(r,\omega) \equiv \frac{E\left[\hat{C}_{xy}(\omega)\right]}{\sqrt{E\left[\hat{C}_{xx}(\omega)\right]}\sqrt{E\left[\hat{C}_{yy}(\omega)\right]}}, \tag{1}$$

where $E[\hat{C}_{xy}(\omega)]$ denotes the ensemble average of $\hat{C}_{xy}(\omega)$ and $r = |\boldsymbol{x} - \boldsymbol{y}|$. For a spatially and temporally stochastic wavefield, the averaged complex coherency coincides with the 0^{th} order Bessel function of the first kind, $J_0\left(\frac{r\omega}{c(\omega)}\right)$ (Aki, 1957; Okada, 2003). Azimuthal averaging is over all $\boldsymbol{x}$ and $\boldsymbol{y}$ for which $|\boldsymbol{x} - \boldsymbol{y}| = r$ and $c(\omega)$ denotes the wave velocity as function of angular frequency.

Spectral whitening of recordings involves normalization of the cross-spectra prior to ensemble averaging. This is mathematically expressed by,

$$\gamma(r,\omega) \equiv E\left[\frac{\hat{C}_{xy}(\omega)}{\sqrt{\hat{C}_{xx}(\omega)}\sqrt{\hat{C}_{yy}(\omega)}}\right]. \tag{2}$$

where γ is dubbed the "whitened complex coherency".

Clearly, expression 2 is different from the one for ρ. Nevertheless, the azimuthal average of γ is used successfully to map subsurface attenuation (Lawrence and Prieto, 2011; Weemstra et al., 2012). These investigators fit a damped Bessel function to the azimuthal average of γ for individual frequencies. They therefore obtain an estimation of the quality factor Q as function of frequency. Their procedure requires multiplication of the damped Bessel function with a factor of proportionality $P(\omega)$ however. Weemstra et al. (2012) find that P generally varies smoothly with frequency but can have values between 0.4 and 0.8.

III. A simple model

We use a very simple (ray-based) model to evaluate the difference between ρ and γ. We assume an isotropic, non-attenuating subsurface and a homogeneous distribution of N sources fixed at locations $\boldsymbol{s}_j$. The imprint of each of these sources at any location $\boldsymbol{x}$ is described by the frequency-domain two-dimensional Green's function, i.e. $\frac{\imath}{4}H_0^{(1)}\left(\frac{r_{jx}\omega}{c(\omega)}\right)$ where $H_0^{(1)}$ is a Hankel function of the first kind of order zero and r_{jx} the distance between source $\boldsymbol{s}_j$ and $\boldsymbol{x}$. The total displacement at $\boldsymbol{x}$ is therefore given by,

$$u(\boldsymbol{x},\omega) = \frac{\imath}{4}\sum_{j=1}^{N} A_j(\omega)e^{\imath\phi_j(\omega)}H_0^{(1)}\left(\frac{r_{jx}\omega}{c(\omega)}\right) \tag{3}$$

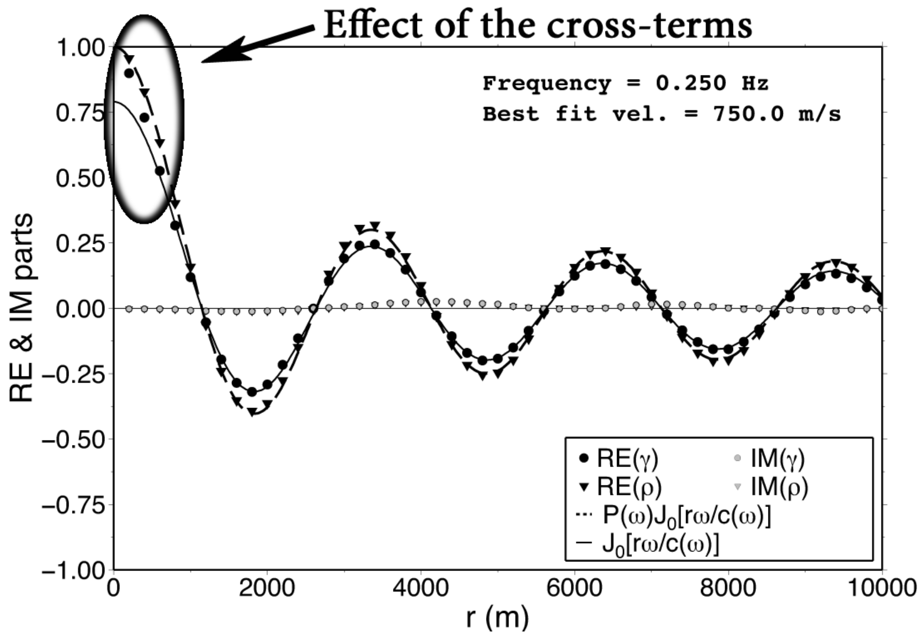

Fig. 1. The real (black) and imaginary (gray) parts of ρ and γ are represented by the solid dots and triangles, respectively. Note that the imaginary parts of γ (gray triangles) are barely recognizable as they are largely overprinted by the imaginary parts of ρ. The Bessel functions that best fit $\Re[\rho]$ and $\Re[\gamma]$ are represented by the dashed and solid curve, respectively.

where the amplitude of the source at $\boldsymbol{s}_j$ is denoted A_j and its phase ϕ_j. We assume the phases ϕ_j to be random. We define a "realization" such, that for a each source ϕ_j is (randomly) different between different realizations. Different realizations are therefore analogous to different time-windows where source phases are assumed to have changed (randomly) from one time-window to the next. Ensemble averages are computed over different realizations.

Because we prescribe a homogeneous distribution of sources, azimuthal averaging becomes redundant and we can simply evaluate the real part of ρ and γ instead of that of their azimuthal average. We compute ρ and γ at intervals of 200 meters for a frequency of 0.25 Hz and prescribe a velocity of 750 m/s. The amplitudes A_j are assumed constant between both different realizations and different sources. Ensemble averages are calculated over 5000 realizations.

IV. Results

Figure 1 shows the behavior of ρ and γ up to a distance of 10 km. A Bessel function is fit to $\Re[\rho]$ (solid line) in the sense that the L1-norm $\left|\Re[\gamma] - J_0\left(\frac{r\omega}{c(\omega)}\right)\right|$ is minimized for varying $c(\omega)$. Similarly, we fit a Bessel function to $\Re[\gamma]$ (dashed line) minimizing $\left|\Re[\gamma] - P(\omega)J_0\left(\frac{r\omega}{c(\omega)}\right)\right|$ for varying $c(\omega)$ and $P(\omega)$. Expectedly, in both cases the prescribed velocity is successfully recovered. More interestingly, the behavior of γ is well approximated by a downscaled Bessel function except for distances shorter than $\sim 1/4$ of a wavelength. We show that this behavior can be explained by the so-called "cross-terms" (which are explained in detail by Wapenaar et al., 2010). This result implies that researchers interested in constraining attenuation using ambient noise surface waves should not include closely spaced stations while fitting a damped Bessel function.

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